at all. To build up this aristocracy, to lay aside gradually the military police, to establish the liberty of the press, to accustom the people to govern and to be governed by each other, to substitute the ambition of civil distinction for that of military glory, to rear sound principles of morals in the place of superstition on the one hand, and infidelity on the other, will demand a long and severe trial, if indeed it do not prove to be impracticable. In the mean time this ill fated country must be content to reap the bitter fruits of past errors, happy if she can ever attain the proud station which her rival has so long enjoyed.

ART. XVII.—1. *An Elementary Treatise on Arithmetic, taken principally from the Arithmetic of S. F. Lacroix, and translated into English, with such alterations and additions as were found necessary, in order to adapt it to the use of the American student.* pp. 128.

2. *An Introduction to the Elements of Algebra, designed for the use of those who are acquainted only with the first principles of Arithmetic. Selected from the Algebra of Euler.* pp. 218.

3. *Elements of Algebra, by S. F. Lacroix. Translated from the French, for the use of the Students of the University at Cambridge, New England.* pp. 263.

4. *Elements of Geometry, by S. M. Legendre, member of the Institute and the Legion of Honor, of the Royal Society of London, &c. Translated from the French for the use of the students of the University at Cambridge, New England.* pp. 208. Cambridge, N. E. Hilliard & Metcalf, 1818—1820.

THESE four volumes form part of a course of pure mathematics, intended for the use of the students at the university of Cambridge. The two first, containing the Arithmetic and Euler's Algebra, are to be studied previous to admission into college; the others form the text books of instruction at the university. A fifth volume has already appeared, containing Trigonometry and Analytical Geometry, and will be followed, we understand, in a few weeks, by a sixth, to contain the usual applications, and complete the course.

It is sometimes objected to the study of mathematics, that it contracts the mind, and, by circumscribing its view, opposes the exercise of invention; that it tends to form a mechanical and skeptical character, rendering the mind incapable of comprehending an extensive subject, and insensible to those nice shades of evidence, and unsusceptible of that accurate perception of beauty and truth, so requisite to quick and fair judgment in matters of taste and morals. This charge, if well founded, would be sufficient to prove this study to be dangerous; and we have no doubt that a belief more or less confident, of its justice, still operates on many persons in prejudice of mathematical pursuits.

It would not be difficult, by reasoning on the nature of the science, and its necessary effects on the mind, to obviate most of these objections. We might mention the many surprising discoveries that have been made in it, and the power of making them, which the study of certain parts communicates, by leading the mind along the natural path of invention. We might show, that if, on the one hand, the view is circumscribed in a particular proposition, while the attention is fixed to a single point; so, on the other, few employments give exercise to a greater grasp and comprehension of mind than the keeping in sight an ultimate object, through all the parts of a long mathematical treatise, and observing the bearing, which each argument and each proposition have on the final one. We might notice too the nature of the investigations in the higher algebra, more abstract and general than the language employed in the metaphysical and moral sciences will admit. In fine, if exposing our weakness could add to our strength, we might point to those parts of analysis, where the path of discovery is so dark, and the mental process so subtle and evanescent, that the most profound masters, while they have with wonder admitted the certainty of the conclusion, have been staggered and divided about the nature, and even the truth of the principles, by which it is attained.

But it is unnecessary to confute by argument, when we have such unbounded evidence from history how groundless are these objections. We may then ask whether all the geometry of Greece could quench or restrain the creative genius and glowing imagination of Plato? Whether the astonishing reach of mind, the proud originality and independence, the faultless taste and eloquent sensibility displayed in the most

perfect writings of his native tongue were diminished by the strong predilection and most complete success with which Pascal cultivated the mathematics? Or has the philosophy of the human mind been investigated with less skill, the parts of this boundless science examined with less patience, the laws regulating the motions of our thoughts noted with less penetration, or the influence of association on the language and imagery of poetry traced with a less delicate pencil, or a less exquisite perception of the beautiful, than if Stewart had not for years discharged the functions of the mathematical professorship at Edinburgh?

It is chiefly with regard to their adaptation to the purpose of instruction, as forming part of a liberal course of studies, that we propose to examine the books before us. In relation to the general question, what should constitute such a course, and in what order the parts should succeed each other, very strong and decided opinions have been expressed, and, we believe, continue to be entertained. The advocate of mathematics often conceives himself obliged to decry the study of the classics, and the lover of classical learning would have that cultivated to the exclusion of science. For ourselves, we think that the study of mathematics might be both introduced sooner, and carried farther, than it is at present, and that to great advantage. At the same time, we think it preposterous to call that education tolerably perfect, which has not been founded on a thorough knowledge of Latin, at least, if not Greek. The primary object of education being the development and cultivation of the powers of the mind, that course of instruction is conceived to be least objectionable, which tends to call forth naturally and exercise strongly all the intellectual faculties; and that person best educated, who, at the completion of his course, has most entire command of them. And we know not that there is any thing, which, in the discipline of the mind, can supply the place of the study of the Latin and Greek. We know of no means to be compared with this for the purpose of communicating the power of quick and delicate discrimination, and of imparting clear perceptions of the difference of words and things. In that part of mathematics which enters into elementary instruction, all is precise and defined. But in language there are so many shades of meaning, differing from each other almost imperceptibly,—the beauty of an expression so often depends on a peculiarity in

the use of one word, or in the arrangement of several,—the distinguishing spirit of an author, especially in a foreign tongue, is so difficult to be perfectly apprehended, and that double process of the judgment, which consists in first ascertaining the meaning of a word in its original connexion, and then selecting the corresponding term in another language, is so continually going on, that all the powers of observation, comparison, and in short, whatever constitutes acquired taste, are constantly called forth and exercised. But while the cultivation of these powers is consigned to their old and tried masters, we should claim that of the power of abstraction and generalization, and much that goes to form certainty and strength of judgment, the habit of persevering research and unbiassed decision, as the just province of the mathematics.

'The study of the sciences,' says Lacroix, 'presents itself under two entirely distinct points of view, either as the means of exercising the mind, of developing the intellectual faculties and rendering them fit for meditation and discussion, or as affording results and precepts immediately applicable to the uses of life and the want of society, which, unfortunately, it does much less frequently than is commonly thought.

'Considered in relation to the first of these objects, which makes it an essential part of education, we perceive the necessity of treating nothing superficially, of diminishing the number of particulars treated of, if necessary, rather than fail to present the subject with the greatest evidence of which it is susceptible, or to render sensible the mechanism of reasoning.'*

Now this end is the only one, with reference to which the study of mathematics is of any great use to the general student; and to the furtherance of this end, the Arithmetic and Algebra of Lacroix are exceedingly well adapted, and a great addition to our means of instruction. The details of science, the rules and results cannot remain long in the strongest memory; and, if they could, they would seldom be of use to the scholar, or the man of business. So, that unless some higher object be effected by the study of the mathematics, the time spent on them will have been thrown away.

Next to this object, of which he never loses sight, some of the rules by which Lacroix seems to have been guided in composing the books before, were; 1°, making use of the analytical method, to pursue, as nearly as possible, the steps of

* Essais sur l'Enseignement, p. 173.

invention; 2°, always to select the most general method; 3°, never to go over the same ground twice, either in his reasoning, or his explanations; 4°, to adapt the elements as he professes to do,* to the great works, which contain all that is most important in science.

The example which he gives are few. He always goes on the supposition that the learner has exercised himself in all the operations and applications of the principles laid down, until he is perfectly familiar with them. 'The choice of examples,' he says,† 'is much more important than their number. A few truths thoroughly comprehended afford greater assistance *to the understanding, in science,* than many theories superficially discussed.'

The few remarks we have to make on the Arithmetic are intended particularly as suggestions to instructers and especially to those parents who have the happiness of instructing their own children. And first, we advise every such person, unless he be learned enough to read French, and fortunate enough to possess Condorect's 'Moyen d'apprendre a compter,' which, by the way, we think one of the most admirable little treatises on arithmetic that ever was written, to possess himself, without loss of time, of this Arithmetic of Lacroix. At the same time, we would give him distinctly to understand, that he is to purchase it for himself and not for *his* children, as for children, young children at least, it was never intended, and to them it is not suited. He must then study a part of it until he is sure he understands it; and we venture to assert, that he must be right well learned, or of a marvellously good understanding, who will not find much light thrown on the dark points in the art, by this little book. Let him, in the next place, when he feels himself thoroughly possessed of the substance of our author's observations on a particular rule, lay aside the book, and, with slate in hand when necessary, and all the eloquence of voice and gesture that he can summon, go through the explanation, and that, for safety's sake, in the very order in which he has learnt it; and, very probably, when he shall have finished, he will himself know much more about it than when he began. After this, looking on the examples in the book only as specimens of what are to be given by himself, let him go on to give example after example. Every thing that is bought in the market, or bar-

* Essais, p. 183. † Ibid. p. 173.

gained for at milliner's, whatever strikes the eye or reaches the ear of a child, must be pressed into this important service. What we have here said, we especially and very warmly recommend to the notice of female instructers, whether the natural or intellectual mothers of their pupils. We profess ourselves sincere lovers of the mathematics, but we believe ourselves disinterested, and loving them only because they help on the process ef education, in its different stages, better than any thing else we know of; and we should be delighted if what we say should induce one mother to inspect this part of her children's education.

What is most deserving of notice in this book, is the manner in which fractions are introduced. They are naturally derived from division; this operation and multiplication are treated of first. Thus the difficulties, which resulted from considering them in a wrong point of view, are avoided, and apparent absurdities removed by new definitions of multiplication and division.

The roots are very properly thrown forward upon algebra, as the explanation that can be given of these rules considered arithmetically, could hardly be made intelligible to a beginner. In the translation a new form is given to reduction and some other parts, better suited to instruction in this country; while the omission of some unusual processes at the end of the book does not diminish its value.

Next in order to the Arithmetic, succeeds the Algebra of Euler. In several respects, it is extremely well suited to elementary instruction, and approves the selection made by the professor. It is divided into short chapters, affording convenient resting place for the impatient and quickly wearied pupil. In another respect, it is better suited to the purpose, especially when we consider the state of instruction among us, than perhaps any other that could be found. It is full of explanations of the little difficulties that are perpetually occurring to a beginner, and of lucid and well selected examples. Many of the explanations, indeed, as of the doctrine of plus and minus, of the process for extracting the roots when applied to numbers, and a few others, are not so good as have since been given, but still much better than most of those that were before accessible to the student.

We know that Euler was, in many places, received with no great respect; sometimes, indeed, he was treated with abso-

lute incivility. Men were ashamed and vexed at not understanding, without study, a book professedly designed for beginners, and wisely determined to lay all the blame upon the book. We admit that the introduction of Euler, or Lacroix, or any other treatise of the kind, must really much increase the labor of an instructer. It is undoubtedly harder, and costs more time and thought to explain a rule to your pupil, than to make him take it for true, without explanation, upon your word and honor, or even because it is in the book and produces the right answer. And we can hardly blame the schoolmaster, into whose motives a desire for the actual improvement of the mind of his pupil does not largely enter, for being unwilling to give up that laudable and philosophical maxim, which, from time immemorial, seems to have commonly prevailed, that algebra was a sort of mystery, a thing to be believed and done, and not inquired into, any more than the gravity of impenetrability of matter.

But, seriously, whatever impression Euler's Algebra may have made on others, we can never forget the feelings with which we at first read it ourselves. It so dissipated the mists left upon the mind after wandering in the obscurity of the English algebraists and their followers, so fairly removed a thousand little obstacles we had always been stumbling at, and threw such light upon the relation and connexion of the whole, that, in the excess of boyish satisfaction, we likened it the Odyssey of Homer, or the first book of Euclid, which a person of taste could never cease to admire, from the time he began to understand. If every other part were only tolerable, the book would deserve to be highly valued, were it but for the chapters on the calculation of irrational quantities, and on the nature of equations of the second degree.

The selections are made from an English translation, which happened to have been previously made, very unfortunately for us, as it deprived us of one from professor Farrar. The professor, too, probably from the engagements consequent on his numerous duties, trusted the correction of the work to some person, among whose qualifications an ignorance of algebra would seem to have held a conspicuous place. This explains the appearance of the numerous errors, and those not merely of the press, which disfigure many of the last pages.*

* In the second edition, printed in 1821, and since the above was written, we understand that most of these errors are corrected.

It may seem a redundancy in the plan, that there are in this course two separate treatises on algebra. But it is to be remembered that they are intended for entirely different classes of students. The multiplication of examples in Euler, and the shortness and simplicity of the explanations, fit it, in a peculiar manner, for beginners. Whereas the philosophical arrangement, the intimate connexion and subordination of all the parts, the mode of instruction by a purely analytical process, and by formulæ perfectly abstract and general, adapt Lacroix only to students of a higher class, and of somewhat cultivated minds. Those, indeed, who have already learnt the mechanism of algebra, will be most likely to seize readily and with satisfaction 'the spirit of the methods' of Lacroix.

We have already mentioned some of the principles by which this author seems to have been directed. He endeavors always to take the reader along with him, never to lay down a rule until it begins to be anticipated, never to give a new process, or bring forward a new principle, until its necessity is felt. We are thus enabled to keep pace with him and be present at his discoveries, to divine his reasons and partake of his power.

The Algebra is divided into two nearly distinct parts, one relating to equations, the other to proportion and progession, including logarithms and questions of interest.

The author begins by solving, in common language, and at length, some simple problems leading to an equation. This furnishes occasion for explaining the use of letters and signs in the place of language. The signs are thus at once fixed in the memory, their utility made evident, and the object of algebra made known. Every thing is introduced in its natural order, and seems to be the consequence of an effort of invention arising from the exigence of the case. All the apparent absurdity of the usual statement of the doctrine of plus and minus both in addition and multiplication is entirely avoided; and the general and abstract nature of algebraical quantities, which is with such difficulty understood by learners, and not understanding which, we have observed to be at the bottom of the obscurity that many persons find in the elements of algebra, is clearly pointed out.

On the subject of fractions, the artifice in the investigation of the common divisor, of introducing into, or rejecting from one of factors any quantities, not found in all the terms of the

other, is exceedingly well illustrated. After some examples, tending to explain the nature of negative, of infinite, and of indeterminate quantities, when they occur in a result, and throwing much light on the metaphysics of algebra, what relates to equations of the first degree is concluded by formulæ for their solution, stated in as mechanical a form, as the warmest advocate for the old fashion could desire.

Introducing equations of the second degree by a question which leads to such an equation, and proving the necessity of some method of extracting roots, he proceeds immediately to the application of the formula $a^2 + 2ab + b^2$ to numbers. This application, and that of the formula for the cube and for other powers, are among the most excellent things in the work, on account of the perfect explanation they afford of the arithmetical processes, an explanation which was hitherto almost wanting.

After giving some methods of approximating the indeterminate roots of numerical quantities, he comes to the common formula for equations of the second degree. In showing, from the nature of the question, the necessity of the roots being imaginary in an equation, of which the known term is negative in the second member, and greater than the square of half the coefficient of the first power of the unknown quantity, he gives a striking specimen of his way of investigating 'analytical facts,' and displaying the spirit of the analysis. One unaccustomed to these investigations is surprised to find himself engaged in examining a general principle, when he thought himself employed on a particular case. Indeed, Lacroix never neglects an opportunity of showing the nature and the power of the instrument he employs. Leaving to the instructer or the reader the application and illustration of principles, he is always hastening on to the development of some new fact. Many of these might be pointed out, but we are deterred by the difficulty of expressing them independently of their application. We notice, as remarkable, the demonstration of the binomial theorem; the most complete, most general, and at the same time most difficult perhaps, to understand, that has been introduced into an elementary book; and the manner in which he employs the principle that $x^n - a^n$ is always divisible by $x - a$. On this he makes the theory of equations to hinge, and, by its various applications, introduces an uniformity and continuity of procedure into this part altogether unusual.

In the article immediately subsequent, a general method for elimination in equations above the first degree is given, founded on the method of Euler; and several examples are brought of its application. Its general nature renders it difficulty to understand, but the difficulty vanishes before a few well selected examples, at least all but the inherent difficulties of a part of algebra, in which so much remains still to be done. We have here some propositions demonstrated in the synthetical form, one of which may serve as an instance of the difference in the mode of demonstration used by the English and French mathematicians. It is proved, that 'if the coefficients of an equation be whole numbers, no one of the roots can be fractional.' It is in the way of *reductio ad absurdum*, but not in the categorical fashion; the consequence being shown, the absurdity is left to be inferred. Now, Wood,* in concluding the same proof, says *outright*, 'that is, $\frac{a^n}{b}$, a fraction in its lowest terms is equal to a whole number, which is absurd; therefore $\frac{a}{b}$ is not a root of the equation.' The whole of this conclusion, in the present and similar cases, is omitted by the French, and to us accustomed to the formality of logic, the proof is apt to seem elliptical.

In giving some general methods of finding the commensurable roots or factors, Lacroix deviates from his usual plan of following the march of invention. Indeed, in a subject so abstruse and extensive as this, it would require more time and space to follow it, than, in an elementary work, could be admitted. The methods, however, are usually illustrated by examples. He then shows how to obtain proximate roots of numerical equations by Newton's method of successive substitutions, as improved by Lagrange.

The second part is remarkable for its simplicity and for the regular gradations by which the doctrine of progressions, ending in diverging series, is deduced from ultimate truths. Logarithms, ways of calculating which he points out, and the mode of employing which, he teaches, are introduced as a part of the solution of the question, to find the value of z in the equation $b^z = c$. They are applied to the solution of questions in interest. One of the formulæ obtained for this purpose will serve at once to show the advantage which would be

* Wood's Algebra, art. 275.

gained in this part of arithmetic, by the employment of algebraical processes, and as an example of our author's method. $A = a(1+r)^n$ is the formula or rule by which either the principal, the amount, the rate per cent. or the number of years, may be found in any question of compound interest, where the three others are known. This performs, by means of two or three simple operations, the work of exceedingly long and complicated arithmetical calculations. The form in which it is applied to each case is given; but when a similar formula has been found for the case in which a new sum is deposited at the end of every year, it is left to be varied by the learner.

What we consider most remarkable and of greatest value in Lacroix's Algebra, are its natural method, the light it throws on the logic of mathematics, and its completely analytical form.

In his method he follows Clairaut, who was the first to compose elements according to the very philosophical conception of introducing the artifices and making the parts succeed each other in the same order in which they might be supposed to have occurred to an original inventor. In the execution of this plan, which was sanctioned by the approbation of Laplace, Lacroix has made several improvements.

In regard to its subserviency to logic in developing the processes of reasoning, he seems to have thoroughly imbibed the views of Condillac. In this respect, his book stands alone. The sole, or certainly the principal object, in other elementary treatises, seems to have been to make expert calculators. Without neglecting this, or, we should rather say, performing it better than it could otherwise be done, he has effected another, and to the general student, we repeat, a very much higher object.

The treatises commonly in use are not strictly analytical. They are algebra delivered in the synthetical form. Of such we have had enough; and we should think it not unimportant, even as a matter of curiosity, that there should be in a course of liberal study, at least one example of the instrument which Newton and Laplace have employed in their sublime discoveries. The form of an elementary treatise, on nearly every other subject, must almost of necessity be synthetical; on analysis itself, it should certainly be analytical. The object of books on other subjects is to communicate truths that are known. On this, at least, it should be to furnish the

means of discovering new. But, 'though truths can be well conveyed in the synthetical way, the method of investigating truths are not communicated by it, nor the powers of invention directed to their proper objects.'*

A skilful analysis is the guide, which must lead to all the discoveries that are to be made in the mechanic arts, in natural philosophy, in medicine, in short, in whatever is to be advanced by deductions from established principles. The synthetical process always supposes the truth known. It cannot go forward a step in the path of discovery. This is the proper work of analysis. And it cannot be a matter of small moment, if we regard the progress of the sciences, whether the student shall or shall not be furnished with a model of the instrument he is to use, in the improvement of the arts of life, and the sciences that contribute to adorn and elevate his existence.

When to these considerations we add, that the book before us forms the first step of a direct introduction to the excellent treatises on astronomy and the various branches of physics, that have proceeded from the hands of the French philosophers, more plain and interesting, as a general characteristic, and, in some instances, incomparably more perfect, than those written on the other side of the British channel, and observe that these admirable bodies of science have hitherto been sealed books to the American student, in consequence of his ignorance of the mathematical language in which they are written, we cannot but consider the translation and publication of these elements as an important era in the history of instruction among us. It is one step, and a very considerable one, towards removing the reproach, to which, from community of language, we have been obnoxious, together with the English, of being almost a century behind the rest of the world in all that relates to mathematical and physical science.

If there be a text-book in use at our colleges more unexceptionable than any other, it is certainly Playfair's Euclid; and yet, some advantages would, we think, be gained, by the substitution of Legendre. Euclid's arrangement, though sometime exceedingly beautiful, is often arbitrary. Parts are separated which belong together. Modes of demonstration are employed, which have become unnecessary and yielded to simpler ones. The whole has the appearance of being in-

* Playfair's Diss. 2d, p. 35.

tended for the entertainment of speculative men, rather than for practical application to the purposes of life. But the catalogue of faults, in conception and execution, is exceedingly small. It is not because Euclid is not admirable, that we would have his place supplied. It is chiefly because he does not give us principles which have been discovered, and methods that have been invented, centuries since his death.

Still, Euclid must always be admired and studied by every lover of science or of argument. If we were challenged to produce a work more perfect in its arrangement than any other we have ever seen, we should, without hesitation, fix on Euclid's First Book. We may imagine, that, on sitting down to compose his elements, he thought first of laying before us the truth contained in the 47th proposition. He arranged all the principles on which it rests, so that every one should stand in its place, and each point directly to the truth he had in view. He never wanders from his purpose. He only steps aside, occasionally, to give an extension, or the converse, or a striking application of the principles he found in his way; but without any more interruption to the unity of his plan, than are the episodes in an epic poem. The thirty-second and forty-third propositions seem to be the only exceptions to these observations; and of these, the former was sufficiently important to be a subordinate object, and of the latter, he foresaw that he should have immediate use in the next book. So far, he left little for future geometricians but to follow. Some of the demonstrations have been simplified, and some things have been demonstrated anew. But still we are doubtful whether the same truths have been, or can be presented in a more natural, or more striking order, or with greater force, than they were by Euclid. But farther than this we cannot go. Most of the propositions in the very next book are valuable only as striking instances of the truth of principles, which we arrive at more readily, and state more clearly, in the simple and comprehensive language of Algebra. Here, too, he is obliged to give us a proof of the imperfection of his instruments, by delaying many of the properties of parallelograms, until he shall have laid down the doctrines of ratios, a doctrine, which belongs no more to geometry than to arithmetic; as it consists of truths that are applicable to all modifications of quantity, and properly makes a part of that language, by which we investigate all.

Legendre's Elements are divided into two parts, the first in four sections on plane geometry, the second in four on solid geometry.

The first section is called 'First Principles, or the properties of perpendicular, oblique, and parallel lines.' On the same subject as Euclid's First, it contains several propositions not to be found there, besides the demonstration, that 'all right angles are equal, parallel lines are throughout at the same distance,' and 'straight lines cannot coincide in part, without coinciding altogether;' all of which Euclid considered either as self-evident, or as consequences of his definitions. Many of the axioms, and all the postulates are omitted. Euclid's fifth is made perfectly easy, by being deduced from the equality of triangles with three equal sides; the seventh is dispensed with; and the demonstrations, especially the indirect ones, of many others, are exceedingly simplified. The foundation of the theory of parallel lines, the plague and shame of geometricians ever since the days of Euclid, is laid on a mechanical construction, which has at least the merit of pointing out to the beginner just where the weak point is, and not leaving him to wonder, on being for the first time *told*, that there is a want of rigor in the structure of the first book of the Elements.

The second section, 'Of the Circle, and the measure of angles,' contains propositions similar to those in the third book of Euclid. In this section he proves the proportionality of angles and arches; and great advantage is derived, throughout the work, from introducing, thus early, a measure which is the last result of what is commonly made the last book of Euclid.

'The third section, entitled Proportions of Figures, contains the measure of surfaces, their comparison, the properties of a right-angled triangle, those of equiangular triangles, of similar figures,' of chords, secants, tangents, and of inscribed figures. Here are propositions like many in the first, second, third, and sixth books of Euclid, with several additional ones. The eleventh in this section is the forty-seventh of Euclid; and to give an instance of the simplicity of our author's arrangement, we observe, that the demonstration, which, in Euclid, rests on scarcely less than thirty propositions, depends here on not more than ten. Even in Dr Thomas Young's Abstract of Mathematics, concise as that is, it rests on several more.

'The fourth section treats of regular polygons, and the measure of the circle. Two lemmas are employed as the basis of this measure, which is otherwise demonstrated after the manner of Archimedes. We then have two methods of approximation for squaring the circle.' The first of these is that of James Gregory, and is this. The surface of two regular inscribed and circumscribed polygons being given, by means of them, polygons, about and within the circle, of double the number of sides, are estimated, and the operation continued, until the surfaces of the two polygons do not differ for a certain number of decimals. This value is taken for the surface of the circle between them. By the second, a square is changed into an equivalent octagon, this into a figure of sixteen sides, &c. and circles are successively inscribed and circumscribed, until their *radii* differ as little as we please from equality. After this section, follows an appendix of ten propositions, on the *maxima* of isoperimetrical figures, ending with a demonstration, that the circle is greater than any other figure of the same perimeter.

The former sections of this part are followed by problems on the application of the principles contained in them. Some of these are very important, as those teaching to find the ratio of commensurable straight lines and angles, and one which attempts to find the common measure of the diagonal and side of a square. This first part contains upwards of fifty new propositions.

Of the Second Part, the first section, 'Of Planes and Solid Angles,' contains several new propositions, and ends with problems for finding a plane and solid angle, from given plane angles and their inclination.

In the second section, 'Of Polyedrons,' are many propositions not found in any other elementary work. To avoid the objections to which the definitions usually given of similar polyedrons are liable, they are described to be those, whose bases being similar, have the vertices of their homologous solid angles determined by triangular pyramids similar, each to each. This is a great improvement; but, in applying the condition thus introduced, he gets into some very long demonstrations, which, it strikes us, might be rendered much more concise. One of the most curious of the demonstrations is that of the solidity of a triangular pyramid. A lemma is given, from which as corollaries are deduced, that such a pyra-

mid must be greater than the fourth, and less than the half of the product of its base by its altitude. These two *limits* being fixed, a proposition follows, in which any supposition, but that of its equality to a third of its base by its altitude, is shown to lead to its being greater than the half, or less than the quarter of this product.

Section third treats of the Sphere and of Spherical Triangles. The whole of this is an addition to the elements. In it some of the properties of spherical triangles are demonstrated, and a measure fixed for the surface of these triangles, of spherical polygons, and lunary surfaces; by this last is meant that surface comprehended between two semicircumferences of great circles, which terminate in a common diameter.

'The fourth section treats of the three round bodies, which are, the Sphere, the Cone, and the Cylinder.' Most of the propositions of this section are demonstrated by the method of Maurolycus. Two bases are supposed, and a regular polygon to be circumscribed about the inner, so that its sides shall not meet the circumference of the greater. On this, if the case require, a regular polyedron is supposed to be constructed, and it is shown that we cannot suppose the measure assigned for the solid or the surface under consideration, to be that of one greater or less, without involving the absurdity, that a figure, contained within another, may be the greater. All these indirect demonstrations are long, and become somewhat tedious from their very great similarity.

It is usually considered necessary to unity of design, in this part of geometry, that an author should employ the same single method of demonstration in all the cases to which it can apply. But why is it not a subordinate object of some importance in an elementary work, to present the reader with a variety of modes of demonstration? In this view, it would be an improvement to have some of these propositions demonstrated by the method of exhaustions, in the concise manner in which it is used by Lacroix. After some more strict mode of demonstration, we might also apply that called the method of indivisibles; and with great advantage to the memory, it establishes such simple and striking relations between the propositions to which it is applicable. The relation between the triangle and parallelogram, for instance, is one of the simplest in geometry; the circle, considered as made up of an infinite number of triangles, whose vertices are at the centre and bases in the

circumference, is referred to the same measure. The same principle might be applied to round bodies. The measure of the solid triangular pyramid being given, the sphere, conceived to be composed of an infinite number of such pyramids, with their vertices at the centre, would immediately be seen to have for the measure of its solidity a third of the radius by its surface. And, as to that surface, if we conceive it to be generated by the revolution of the semiperimeter of a polygon, of an infinite number of sides, about a diameter, we have for its measure that diameter multiplied by a circumference. Many applications will occur to the geometrician; and, if it be objected to the strictness of the method, that it involves the consideration of infinity, we might answer, that this consideration enters into all the other modes of demonstration, that it always has entered, and, for aught we see, always will enter into those demonstrations in which curved lines are referred to straight lines. And it matters not whether this idea of infinity is presented in the form of the indefinite approach of two curves, with a polygon between them, of the unlimited multiplication of the sides of two rectilineal figures, or the infinitely numerous sides of one.

From the uniform simplicity of the demonstrations of the first part, it is exceedingly well adapted to elementary instruction, and might be introduced immediately after Euler's Algebra. The difficulty of the second makes it best to have it put off to a more advanced part of the course. Indeed, the easier parts of geometry might be introduced much earlier in the course of instruction than they usually are. Geometry addresses itself more immediately to the senses; and the synthetical demonstration is more strikingly and irresistibly convincing than perhaps any other; and one reason that it has not been earlier introduced is, doubtless, the difficulty of some of the first propositions in Euclid, the universal text-book. This difficulty is now removed, and it is one of sufficient magnitude to authorize a considerable change. It seems important that mathematics should be introduced as early as possible in the course, in order that the powers which it exercises may sooner be brought into complete action, and the time afterwards be better employed in those studies which are of more immediate practical utility, and whose object and tendency is rather to furnish the mind, than to give it strength.

The translator has introduced, under the title of an intro-

duction to the geometry, an explanation of the algebraical signs, and the theory of proportions, taken, with improvements, from Lacroix's Geometry. A small part of the original of Legendre is omitted. The only portion we regret, is a few propositions on regular polyedrons, which are very simple, and would be likely to interest beginners.

For the convenience of the student, the plates of this volume are separate from the volume itself. The whole work is executed with great care. It is rare to find a mathematical book, from the English or French presses, so uniformly free from errors.

Art. XVIII.—*Poems by William Cullen Bryant.* Cambridge, Hilliard & Metcalf. pp. 44.

Of what school is this writer? The Lake, the Pope, or the Cockney; or some other? Does he imitate Byron or Scott, or Campbell? These are the standing interrogatories in all tribunals having the jurisdiction of poetry, and it behoves us to see that they are administered. He is then of the school of nature, and of Cowper; if we may answer for him; of the school which aims to express fine thoughts, in true and obvious English, without attempting or fearing to write like any one in particular, and without being distinguished for using or avoiding any set of words or phrases. It does not, therefore, bring any system into jeopardy to admire him, and his readers may yield themselves to their spontaneous impressions, without an apprehension of deserting their party.

There is running through the whole of this little collection, a strain of pure and high sentiment, that expands and lifts up the soul and brings it nearer to the source of moral beauty. This is not indefinitely and obscurely shadowed out, but it animates bright images and clear thoughts. There is every where a simple and delicate portraiture of the subtle and ever vanishing beauties of nature, which she seems willing to conceal as her choicest things, and which none but minds the most susceptible can seize, and no other than a writer of great genius, can body forth in words. There is in this poetry something more than mere painting. It does not merely offer in rich colours what the eye may see or the heart feel, or what may fill the imagination with a religious grandeur.

ERRATA IN THE LAST NUMBER.

Page	86	*line*	4 from bottom	*for*	'to it,'	*read*	'it to.'
"	88	"	16 from top	"	'West Florida,'	"	'Spain.'
"	97	"	31 " "	"	'Trumbull,'	"	'Turnbull.'

IN THIS NUMBER.

Page 305 *line* 6 from top *for* 'vaccinated,' *read* 'inoculated.'
" 370 " 3 from bottom *read* 'the,' *before* 'factors.'
" 431 Highway robbery, under certain circumstances, is to be added to the list of capital crimes in Massachusetts, by a late statute.